

Introduction: Combustion instability in solid rocket motors and liquid engines has long been a subject of concern. Many rockets display violent fluctuations in pressure, velocity, and temperature originating from the complex interactions between the combustion process and gas dynamics. Recent advances in energy based modeling of combustion instabilities require accurate determination of acoustic frequencies and mode shapes. Of particular interest is the acoustic mean flow interactions within the converging section of a rocket nozzle, where gradients of pressure, density, and velocity become large. The expulsion of unsteady energy through the nozzle of a rocket is identified as the predominate source of acoustic damping for most rocket systems. Recently, an approach to address nozzle damping with mean flow effects was implemented by French [1]. This new approach extends the work originated by Sigman and Zinn [2] by solving the acoustic velocity potential equation (AVPE) formulated by perturbing the Euler equations [3]. The present study aims to implement the French model within the COMSOL Multiphysics framework and analyzes one of the author's presented test cases.

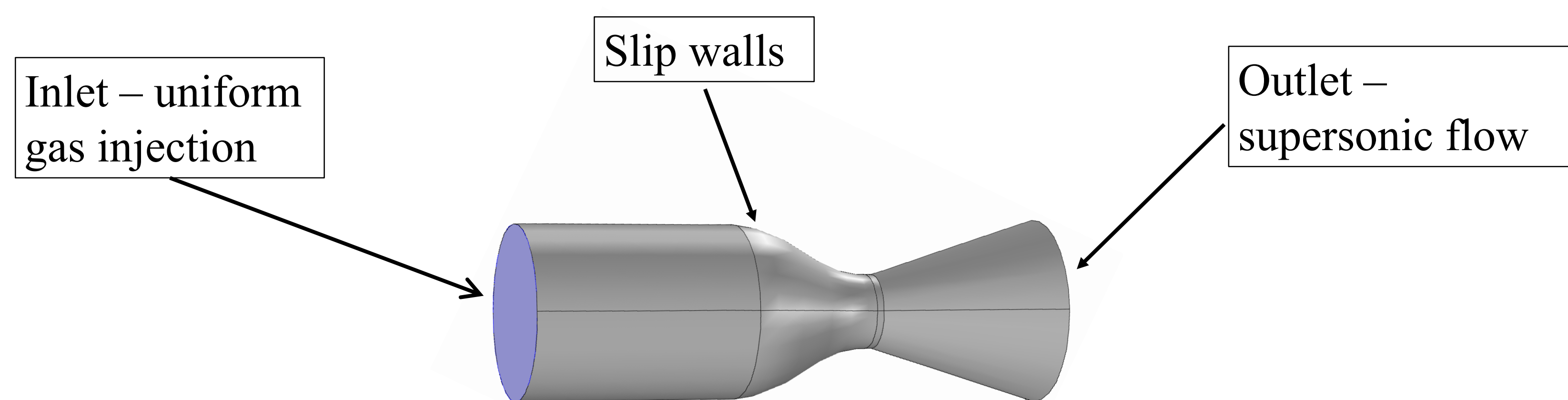


Figure 1. Test case geometry of a simulated liquid rocket engine.

Computational Methods: The AVPE is utilized to model linear non-dissipative acoustic waves in the presence of an isentropic inhomogeneous mean flow. [3]

$$\nabla^2 \psi - (\lambda/c)^2 \psi - \mathbf{M} \cdot [\mathbf{M} \cdot \nabla(\nabla \psi)] - 2(\lambda \mathbf{M}/c + \mathbf{M} \cdot \nabla \mathbf{M}) \cdot \nabla - 2\lambda \psi [\mathbf{M} \cdot \nabla(1/c)] = 0$$

c = Speed of sound

\mathbf{M} = Mach vector

$\lambda = \alpha + i\omega$, Complex eigenvalue

$\psi = \psi^r + i\psi^i$, Complex acoustic potential

Process:

- Utilize HMNF module to calculate mean flow properties within chamber.
- Post process CFD result to extract the sonic plane, $M=1$.
- Import sonic plane into geometry as a natural boundary for the AVPE.
- Define source terms in the Coefficient Form PDE module.
- Use eigenvalue solver to determine the complex λ and ψ .

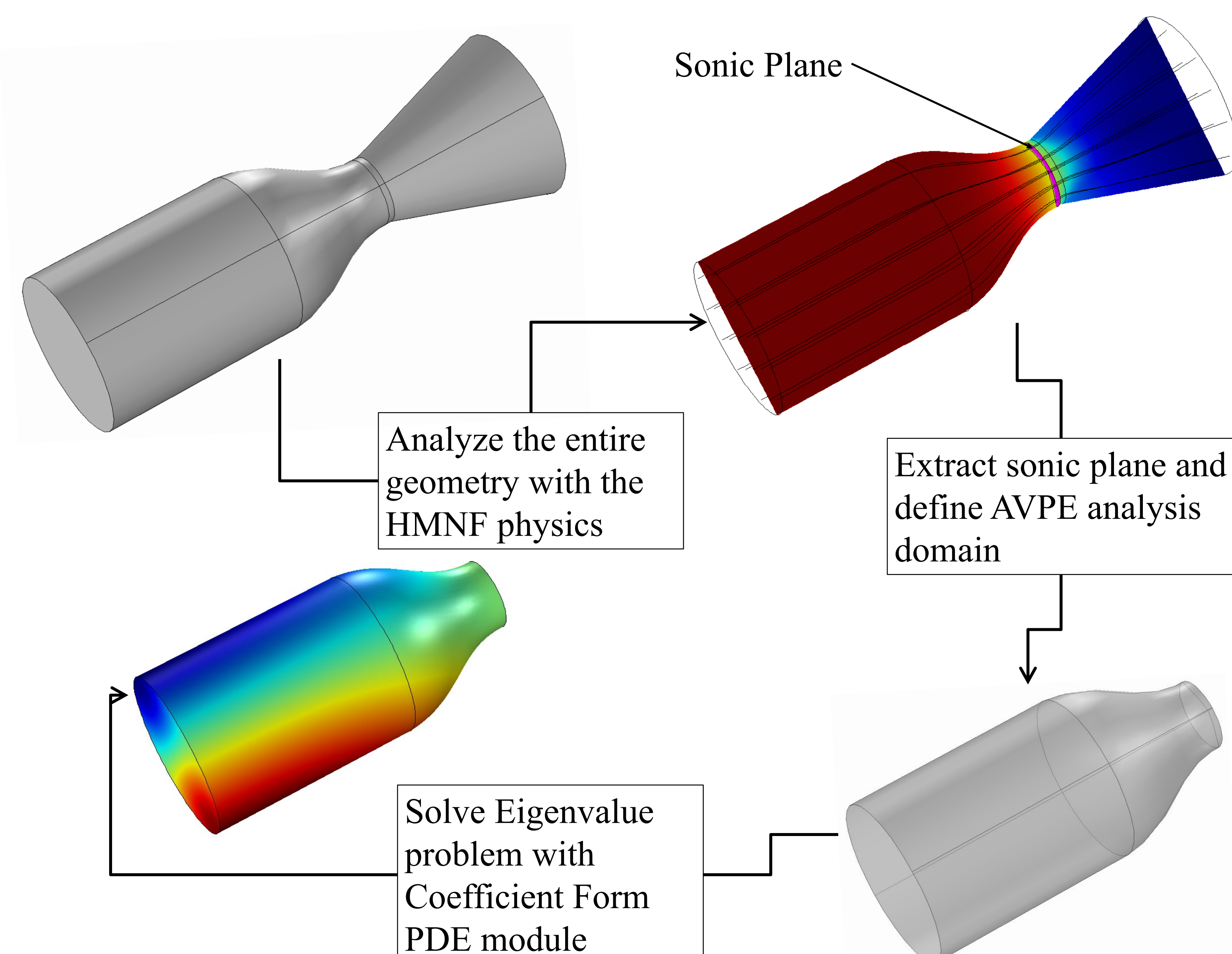


Figure 2. COMSOL implementation and solution flow diagram.

Results: The Eigenvalue solver produces complex Eigenmodes and Eigenvalues representing each acoustic mode and its complex conjugate. Presented in Figure 3 is a reproduction of the acoustic potential mode shape presented by French for the first tangential (1T) mode [1]. Figure 4 displays the 1T acoustic potential mode shape derived from the present work. The contours in Figures 3 and 4 are almost indistinguishable and display the effect of the mean flow on the acoustic wave. Figures 5 and 6 show a comparison of the 1T mode shape derived from the classic homogenous wave equation and the AVPE, respectively, through a half of its period (T).

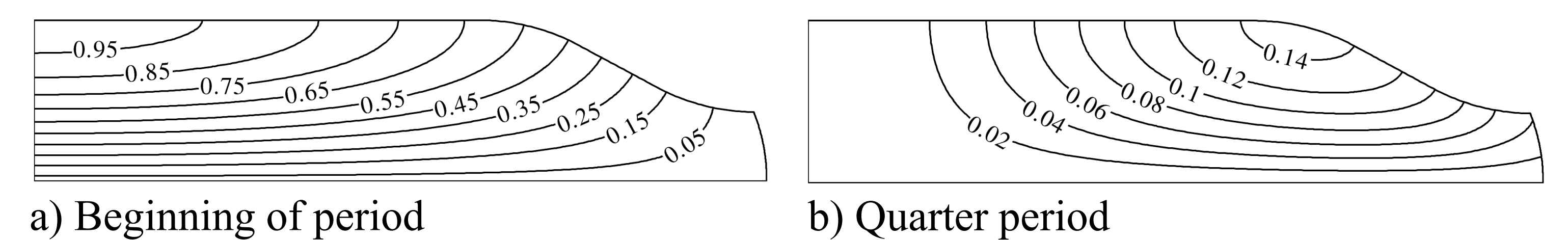


Figure 3. First tangential acoustic potential derived by French. [1]

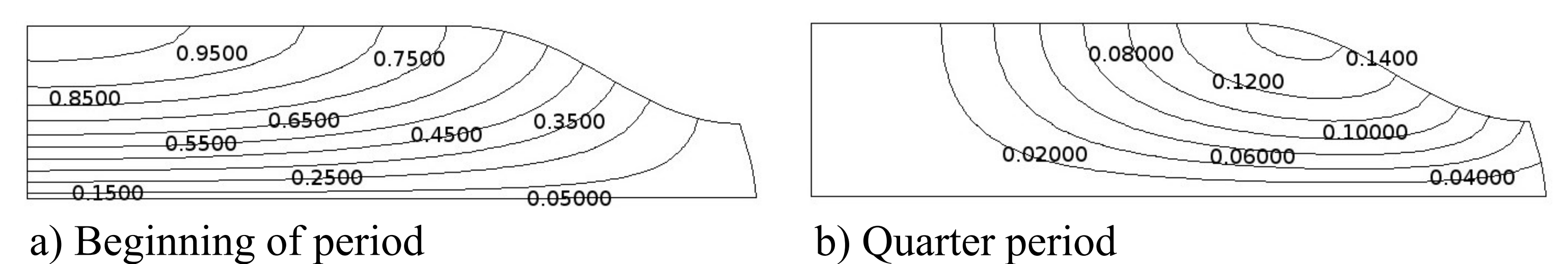


Figure 4. First tangential acoustic potential from COMSOL analysis.

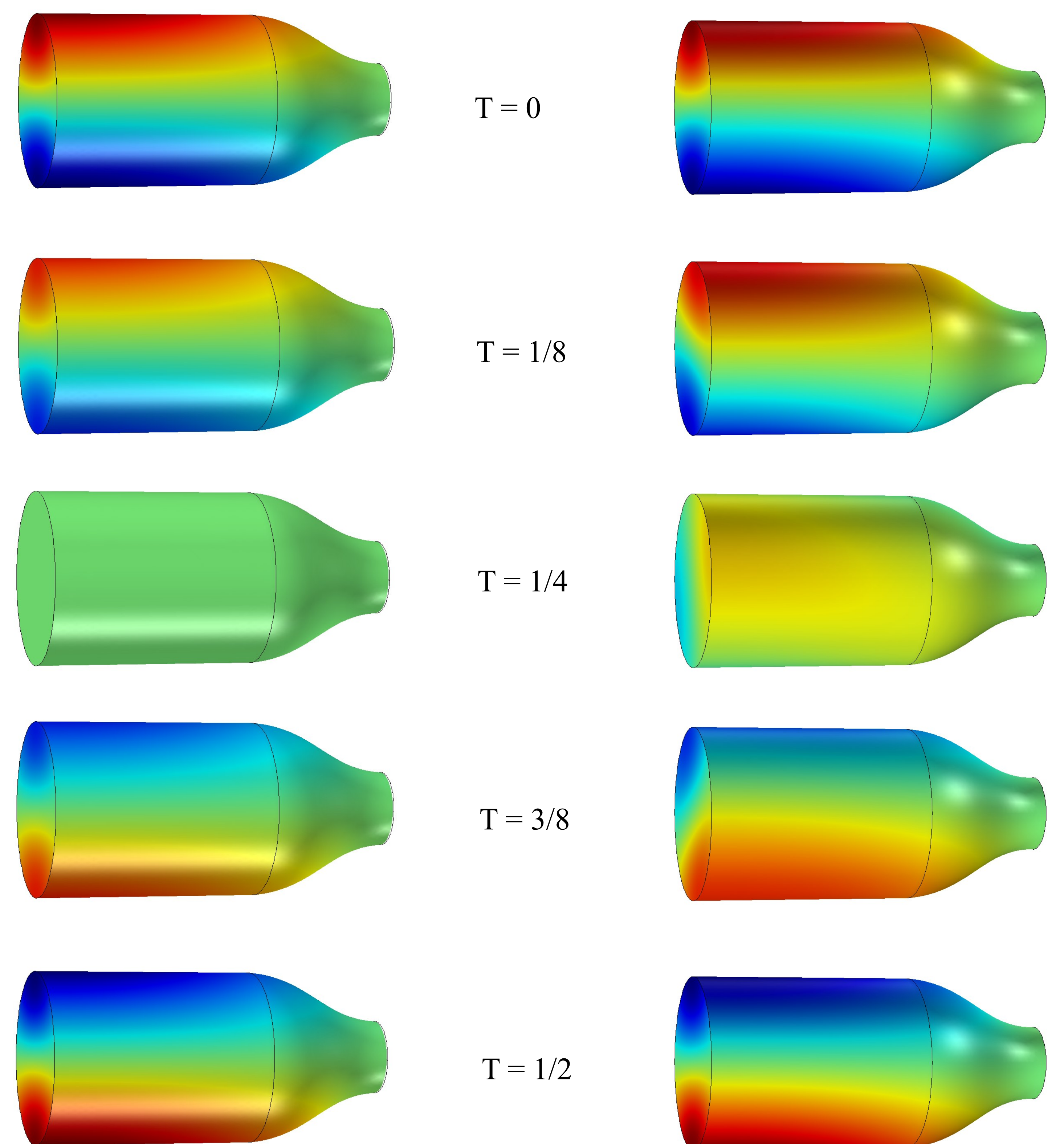


Figure 5. First tangential eigenmode using classic wave equation.

Figure 6. First tangential eigenmode using the AVPE.

Conclusions: The AVPE was solved in a simulated liquid engine configuration using a combination of the HMNF and Coefficient Form PDE modules within the COMSOL Multiphysics framework. Results from the analysis compare well with that of a study performed using the finite volume method. The comparison of mode shapes from the classic wave equation and the AVPE demonstrate the need for this analysis if any accurate combustion instability modeling is to be performed.

References:

1. J. C. French, *Nozzle Acoustic Dynamics and Stability Modeling*, vol. 27, Journal of Propulsion and Power, 2011.
2. R. K. Sigman and B. T. Zinn, *A Finite Element Approach for Predicting Nozzle Admittances*, vol. 88, Journal of Sound and Vibration, 1983, pp. 117-131.
3. L. M. B. C. Campos, *On 36 Forms of the Acoustic Wave Equation in Potential Flows and Inhomogeneous Media*, vol. 60, Applied Mechanics Reviews, 2007, pp. 149-171.